英

令和５年４月入学

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学B　MATHEMATICS B

令和５年２月28日(火)
Tuesday, February 28, 2023　13:00 － 14:30

Notice

1. Do not open this examination booklet until instructed to do so.
2. An examination booklet, answer sheets, draft sheets are provided. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all problems. Use two answer sheets for each problem. Indicate the problem number you chose on the answer sheets.
4. At the end of the examination, reconfirm your examinee number and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = \sin x$

(2) $\frac{dy}{dx} + xy = \frac{x}{y}$

(3) $(x^2 - xy)dx + x^2dy = 0$

2．The Laplace transform of a function $y(t)$ is defined by

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty y(t)\, e^{-st} dt.$$

Solve the following problems. If necessary, use the following equations,

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2} \text{ and } \mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2},$$

where a is a positive constant.

(1) Show $\mathcal{L}\{t\, y(t)\} = -\frac{d}{ds}Y(s)$.

(2) Obtain $\mathcal{L}\{t\cos t\}$ and $\mathcal{L}\{t\sin t\}$.

(3) Obtain the inverse Laplace transform of $\frac{1}{(s^2+1)^2}$ by applying the inverse Laplace transform to both sides of the following equation,

$$\frac{d}{ds}\left(\frac{s}{s^2+1}\right) = \frac{2}{(s^2+1)^2} - \frac{1}{s^2+1}.$$

(4) Obtain $Y(s)$, when $\frac{dy}{dt} + y(t) = t\cos t$ and $y(0) = 1$.

(5) Obtain $y(t)$ in problem (4).